Grading Scheme

IGO 2020

7th Iranian Geometry Olympiad

Elementary Level

Problem 1.

(8 points) Drawing 5 lines or less than that.

- **(4 points)** Drawing 6 lines.
- **(4 points)** Incomplete solutions with 3 or 4 lines (so that the solution can be completed).
- No points will be given to incomplete solutions with less than 3 folds.

Problem 2.

(2 points) Showing that $FA = FC$ and $GA = GC$.

(3 points) Proving that $AFH \cong CFE$.

(2 points) Showing that $FE = FH$ and $GE = GH$.

(1 point) Finishing the solution.

Problem 3.

(4 points) putting up 3 triangles next to each other.

(2 points) Showing that $x + y + z$ is bigger than 2 times each side (for example $x + y + z > 2a$).

(2 points) Finishing the solution.

Problem 4.

(2 points) Showing that $EM \parallel BC$ or $FN \parallel BC$.

(2 points) Construction the point X.

(1 point) Showing that the point M is midpoint of EX.

(2 points) Proving that the line MP passes through the midpoint of BC.

(1 point) Finishing the solution.

Problem 5.

(3 points) First Lemma

(3 points) Second Lemma

(2 points) Finishing the solution

- Proof for infinitely many values of n (not all n) worth at most 5 points

Intermediate Level

Problem 1.

(2 points) Construction the points P.

(4 points) Proving that the points P, M, N are collinear.

(2 points) Finishing the solution.

Problem 2.

(2 points) Showing that $\angle OCM = \angle TAO$.

(2 points) Proving that $OCM \cong TAO$.

(3 points) Proving that $AOC \sim MOT$.

(1 point) Finishing the solution.

Problem 3.

(3 points) Proving that $EK \parallel AC$.

(2 points) Proving that $\angle KJQ = \angle QPC$ or it suffice to show that $CPBQ$ is cyclic.

(3 points) Showing that $CPBQ$ is cyclic.

Problem 4.

(1 point) Construction the points P and Z.

(2 points) Showing that H and J are isogonal conjugate with respect to triangle BPC.

(2 points) Proving that $JEF \sim ZBC$.

(2 points) Proving that the point Z lies on the line HP.

(1 point) Proving that the points X and Y lie on the line HP.

Problem 5.

(1 point) Proving there is no example for odd values of n.

(1 point) The lemma for having a right-tringle in R^3 from a triangle in the plane.

(2 points) Construction of a polygon as in the official solution.

(4 points) Construction of the polyhedron form the polygon together with proof that it has the properties.

- If in the last part the solution for general case needs to investigate small values of n separately and it had not been done 1 point will be deducted.
- Giving example for infinitely many values of n (not all even n) worth at most 4 points (from 7 points of giving examples)
- **(1 point)** Only giving example for finitely many values of n.

Advanced Level

Problem 1.

(2 points) Considering the foot of the altitude AH or drawing the altitude

(2 points) Construction the points Q, K .

(3 points) Proving that the points Q, K, H are collinear.

(1 point) Showing the point H is midpoint of EF .

Problem 2.

(3 points) Considering Homothety with center A and ratio $\frac{1}{2}$ and giving the equivalent statement, the tangency the circumcircle MNT to AI .

(2 points) Proving that SI is tangent to the circumcircle of triangle ITN .

(3 points) Proving that SI is tangent to the circumcircle of triangle NIM .

Problem 3.

(0 point) The internal common tangent of each two circles intersect the third one.

(1 point) Restrictions on the common point of the third circle with the internal tangents.

(4 points) Giving bounds for the radius of the third circle using previous.

(3 points) Final inequalities and finishing the solution

- For any other similar solution, **(1 point)** for geometric arguments, **(4 points)** for bound on the radius of the third circle, and **(3 points)** for inequalities and finishing the solution.
- **(5 points)** Solutions for weaker results with $c \leq 3$.
- **(3 points)** Solutions for weaker results with $c > 3$.
- For solutions based on showing the sharp case is when the circles are excircles of a triangle. **(4 points)** for showing this is the best case, and **(4 points)** for the inequalities in this special case.

Problem 4.

(3 points) Proving the lemma 3

(3 points) Calculating the value of $\sin EIF$.

(1 point) Investigating the case 1.

(1 point) Investigating the case 2.

Problem 5.

- **Solution 1:**

(1 point) Proving the lemma.

(1 point) Considering the homothety between ω_1 and ω_2 .

(2 points) Construction of the point L .

(1 point) Showing that $\angle HJD = 90^\circ$ and the problem statement is true when $K \equiv D$.

(2 points) Showing that L lies on the median.

(1 point) Finishing the solution.

- **Solution 2:**

(1 point) Proving the lemma.

(1 point) Proving that $FMN \sim APJ$.

(2 points) Showing that the points A, F, N' are collinear.

(1 point) Showing that $\angle HJD = 90^\circ$ and the problem statement is true when $K \equiv D$.

(1 point) Proving that $ALHJ$ is cyclic.

(1 point) Proving that $\angle HLN = 90^\circ$.

(1 point) Finishing the solution.